SM3H HW11.4 Eliminating the Parameter

When a curve is defined parametrically, it is sometimes possible to eliminate the parameter and find a way to define the curve with one parameter being a function of the other. While this will result in a function that no longer has a direction based on the original independent variable, the relationship between parameters will remain.

The essential strategy for eliminating the parameter is:
- Solve for the independent variable in one of the parameters.
- Substitute the value found above into the independent variable in the other parameter.
- Simplify in a manner that doesn't cause loss of any portion of the original curve to find $y(x)$.

Example 1: Eliminate the parameter t from the definition of $a(x, y) = < x(t), y(t) >$

$$x(t) = 4t - 1, \;\; y(t) = 2t + 3, \quad -\infty < t < \infty$$

We select $x(t)$ as the parameter to use first so that we can substitute into $y(t)$ to find $y(x)$.		
Step 1: Solve $x(t)$ for t.	$x =$ $x + 1 =$ $0.25x + 0.25 =$	$4t - 1$ $4t$ t
Step 2: Substitute t into $y(t)$.	$y =$	$2(0.25x + 0.25) + 3$
Step 3: Simplify carefully.	$y =$ $y =$	$0.5x + 0.5 + 3$ $0.5x + 3.5$

So, the graph of $y(x) = 0.5x + 3.5$ should contain the same points as the original parametric curve, $a(x, y)$. We'll no longer be able to use t-values to determine the motion along the curve, but we have an easier representation to use for graphing.

Example 2: Eliminate the parameter t from the definition of $a(x, y) = < x(t), y(t) >$

$$x(t) = t^2 - 2, \;\; y(t) = 5t, \quad -\infty < t < \infty$$

We select $x(t)$ as the parameter to use first so that we can substitute into $y(t)$ to find $y(x)$.		
Step 1: Solve $x(t)$ for t.	$x =$ $x + 2 =$ $\pm\sqrt{x + 2} =$	$t^2 - 2$ t^2 t
Step 2: Substitute t into $y(t)$.	$y =$	$5\left(\pm\sqrt{x + 2}\right)$
Step 3: Simplify carefully.	$y =$	$\pm 5\sqrt{x + 2}$
We can't remove the $\pm$ without removing some of the curve!		

When we have t contained in a trig function, it is easier to make appropriate substitutions into $\cos^2 t + \sin^2 t = 1$ than using inverse trig to solve for t. We typically stop simplifying these equations once they are in the standard form for a circle or conic section.

Example 3: Eliminate the parameter t from the definition of $a(x, y) = < x(t), y(t) >$

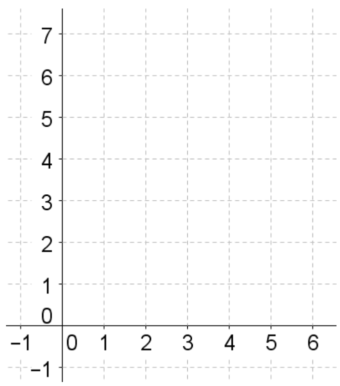
$$x(t) = \cos t\,,\quad y(t) = \sin t\,,\quad -\infty < t < \infty$$

Step 1: Use $\cos^2 t + \sin^2 t = 1$.	$\cos^2 t + \sin^2 t =$	1
Step 2: Substitute x into $\cos t$ and y into $\sin t$.	$x^2 + y^2 =$	1 This is the equation of a circle, so more simplification isn't necessary unless it helps to graph.
Step 3: Simplify carefully.	$y^2 =$ $y =$	$1 - x^2$ $\pm\sqrt{1 - x^2}$

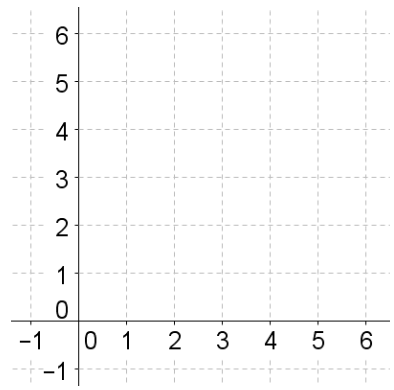
HW11.4

Eliminate the parameter and write an equation for the curve in the form of $y = f(x)$. Then, sketch the curve.

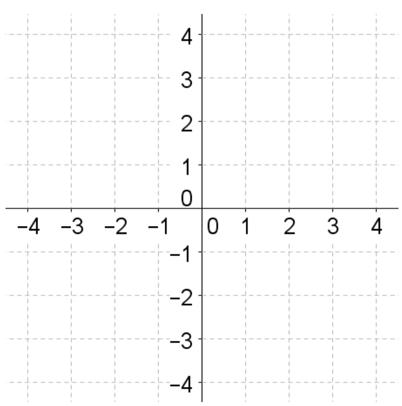
1) $a =< t + 3, 4 - t >$



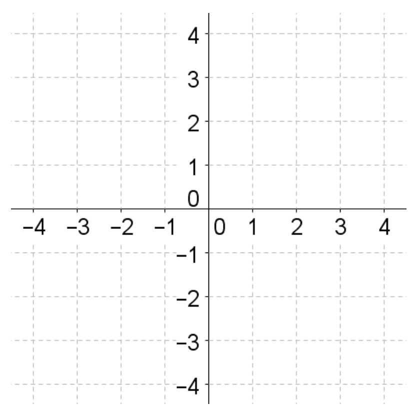
2) $b =< t^2, 3 - t >$



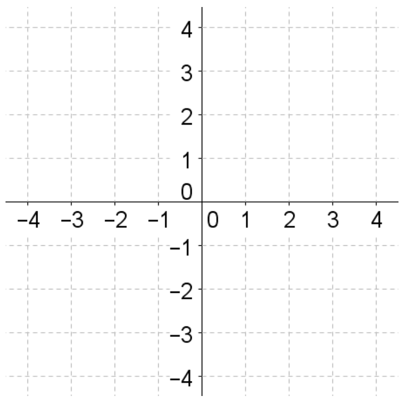
3) $c =< \frac{5}{t}, t - 1 >$



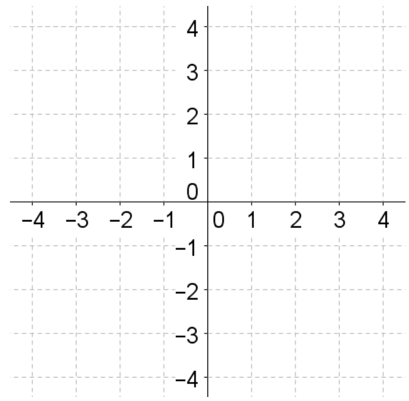
4) $d =< \sin t, \cos t >$



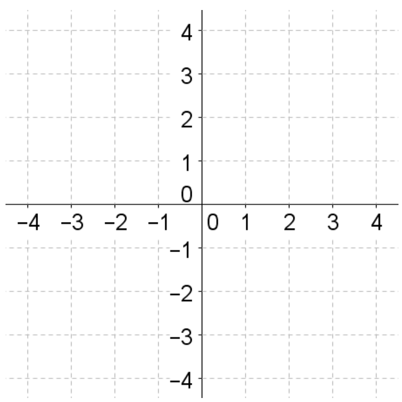
5) $f = < \ln t, 2t >$



6) $g = < \sqrt{t}, \sqrt[4]{t} >$



7) $h = < \cos(t) + 2, \sin(t) - 3 >$



8) $j = < \sin(t) - 3, \cos(t) + 3 >$